Improved Unitary Root-MUSIC for DOA Estimation Based on Pseudo-Noise Resampling

Cheng Qian, Lei Huang, and H. C. So

***Abstract*—A novel pseudo-noise resampling (PR) based unitary root-MUSIC algorithm for direction-of-arrival (DOA) estimation is derived in this letter. Our solution is able to eliminate the abnormal DOA estimator called outlier and obtain an approximate outlier-free performance in the unitary root-MUSIC algorithm. In particular, we utilize a hypothesis test to detect the outlier. Meanwhile, a PR process is applied to form a DOA estimator bank and a corresponding root estimator bank. We propose a distance detection strategy which exploits the information contained in the estimated root estimator to help determine the final DOA estimates when all the DOA estimators fail to pass the reliability test. Furthermore, the proposed method is realized in terms of *real-valued* computations, leading to an efficient implementation. Simulations show that the improved MUSIC scheme can significantly improve the DOA resolution at low signal-to-noise ratios and small samples.**

***Index Terms*—Direction-of-arrival (DOA) estimation, pseudo-noise resampling (PR), small sample size, unitary root-music.**

I. Introduction

MANY applications in radar, sonar and wireless communications require direction-of-arrival (DOA) information of multiple sources impinging upon a sensor array. Numerous DOA estimators have been proposed in the literature. Among them, subspace methods such as ESPRIT [1], [2] and MUSIC [3], offer a good compromise between accuracy and computational complexity. However, this kind of algorithms suffers considerable performance degradation when the number of snapshots and/or signal-to-noise ratio (SNR) is small [4], [5], [6]. The most visual embodiment of this phenomenon is that the observed estimation errors rapidly depart from the Cramér-Rao lower bound (CRLB) below a threshold SNR [5].

Gershman and Böhme [6] have proposed an enhanced weighted MUSIC estimator which can improve the DOA estimation performance via a pseudo-noise resampling (PR) of spatial spectrum. Compared with the conventional MUSIC algorithm, it significantly increases the DOA resolution in low SNR scenario. But it still needs to search the spectrum, requiring huge computational burden which may hinder its practical application. The idea behind the PR technique which is motivated by the success of modern resampling schemes (e.g., bootstrap [7], [8]), is to utilize synthetically generated psuudo-noise to perturb the original noise in such a way that the outlier will be removed [9]. Vasylyshyn [10] has proposed a variant of the PR-based root-MUSIC algorithm which combines the PR technique with the conventional beamformer. In [11], a PR technique based unitary ESPRIT algorithm has been developed to mitigate the effect of outliers. In [12], the author utilizes the PR process and local performance test to improve the accuracy of the beamspace ESPRIT algorithm. However, the ESPRIT-like algorithms suffer performance degradation when the number of overlapping sensors is large.

In this paper, we derive an improved unitary root-MUSIC algorithm that is based on the PR technique to reduce the threshold SNR. Our approach utilizes a PR process to construct a DOA estimator bank and a corresponding root estimator bank. Then a reliability test [6], [10], [11] is applied to the whole DOA estimator bank and only the DOA estimator that has passed the reliability test can be retained. Furthermore, we have proposed a DOA selection strategy to determine the final DOA estimates by replacing the outliers with appropriate candidates in the retained DOA estimators. To combat the case where there is no DOA estimator passing the reliability test, unlike the conventional median average method [10], [11], we propose a distance detection strategy (DDS) which utilizes the information contained in the root estimator bank to determine the final DOA estimates. Due to the real-valued formulation, our proposal enables an efficient implementation when a moderate number of PR processes is given. Simulations show that the PR based unitary root-MUSIC scheme outperforms its counterparts in the low SNR and small sample situations.

II. Problem Formulation and Unitary Root-MUSIC

A. Signal Model

Consider a uniform linear array (ULA) with M isotropic sensors. There are $P(P < M)$ uncorrelated narrowband source signals impinging on the array from distinct directions $\{\theta_1, \cdots, \theta_P\}$ in the far field. The $M \times 1$ observation vector is

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \quad t = 1, \cdots, N. \tag{1}$$

Here, $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_P)]$ is the steering matrix, $\boldsymbol{s}(t) = [s_1(t), \cdots, s_P(t)]^T$ is the source signal vector, N is the number of snapshots and the steering vector due to the pth source is expressed as

$$\boldsymbol{a}(\theta_p) = [1, e^{j2\pi \sin\theta_p d/\lambda}, \cdots, e^{j2\pi(M-1)\sin\theta_p d/\lambda}]^T \tag{2}$$

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C. Qian and L. Huang are with the Department of Electronic and Information Engineering, Harbin Institute of Technology Shenzhen Graduate School, Shenzhen, Guangdong 518055, China (e-mail: alextoqc@gmail.com; dr.lei.huang@ieee.org).

H. C. So is with the Department of Electronic Engineering, City University of Hong Kong, Hong Kong 852, China (e-mail: hcso@ee.cityu.edu.hk).

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where $(\cdot)^T$ is the transpose, λ is the carrier wavelength and $d = \lambda/2$ is the interelement spacing. It is assumed that the noise $\boldsymbol{n}(t)$ is a white Gaussian process with zero mean and covariance $\sigma_n^2 \boldsymbol{I}_M$, where $\boldsymbol{I}_M$ is the $M \times M$ identity matrix. Moreover, the noise is uncorrelated with the signal $\boldsymbol{s}(t)$. The covariance matrix of $\boldsymbol{x}(t)$ is

$$\boldsymbol{R} = \mathbb{E}[\boldsymbol{x}(t)\boldsymbol{x}(t)^H] = \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^{\mathrm{H}} + \sigma_n^2 \boldsymbol{I}_M \tag{3}$$

where $\boldsymbol{R_s} = \mathbb{E}[\boldsymbol{s}(t)\boldsymbol{s}(t)^H]$, $\mathbb{E}[\cdot]$ stands for the mathematical expectation and $(\cdot)^H$ represents the conjugate transpose.

B. Unitary Root-MUSIC Algorithm

The unitary root-MUSIC algorithm [13] utilizes a real-valued covariance matrix given as

$$\begin{aligned} C &= \frac{1}{2}\boldsymbol{Q}_M^H \left(\boldsymbol{R} + \boldsymbol{J}_M \boldsymbol{R}^* \boldsymbol{J}_M\right) \boldsymbol{Q}_M \\ &= \mathrm{Re}\left\{\boldsymbol{Q}_M^H \boldsymbol{R} \boldsymbol{Q}_M\right\} \end{aligned} \tag{4}$$

where $\mathrm{Re}\{\cdot\}$ represents the real part, $\boldsymbol{J}_M$ is an $M \times M$ exchange matrix with ones on its antidiagonal and zeros elsewhere, $(\cdot)^*$ represents complex conjugate and $\boldsymbol{Q}_M$ is a sparse unitary matrix, defined as [11], [13]

$$\boldsymbol{Q}_M = \begin{cases} \frac{1}{\sqrt{2}} \begin{bmatrix} \boldsymbol{I}_l & j\boldsymbol{I}_l \\ \boldsymbol{J}_l & -j\boldsymbol{J}_l \end{bmatrix}, & \text{for} \quad M = 2l \\ \frac{1}{\sqrt{2}} \begin{bmatrix} \boldsymbol{I}_l & \mathbf{0}_l & j\boldsymbol{I}_l \\ \mathbf{0}_l^T & \sqrt{2} & \mathbf{0}_l^T \\ \boldsymbol{J}_l & \mathbf{0}_l & -j\boldsymbol{J}_l \end{bmatrix}, & \text{for} \quad M = 2l+1. \end{cases} \tag{5}$$

Here, $\mathbf{0}_l$ is an $l \times 1$ zero vector. Define the eigenvalue decomposition (EVD) of $\boldsymbol{C}$ as

$$\boldsymbol{C} = \boldsymbol{E}\Lambda\boldsymbol{E}^H = \boldsymbol{E}_S\Lambda_S\boldsymbol{E}_S^H + \sigma_n^2 \boldsymbol{E}_N\boldsymbol{E}_N^H \tag{6}$$

where $\boldsymbol{E}_S = [\boldsymbol{e}_1, \cdots, \boldsymbol{e}_P]$, $\Lambda_S = \mathrm{diag}\{\lambda_1, \cdots, \lambda_P\}$ and $\boldsymbol{E}_N = [\boldsymbol{e}_{P+1}, \cdots, \boldsymbol{e}_M]$ with $\{\lambda_i\}_{i=1}^P$ being the signal eigenvalues, $\{\boldsymbol{e}_i\}_{i=1}^P$ being its corresponding signal eigenvectors, $\{\boldsymbol{e}_i\}_{i=P+1}^M$ being the noise eigenvectors and $\mathrm{diag}\{\cdot\}$ being a diagonal matrix. Then the unitary root-MUSIC polynomial can be expressed as

$$f_{\mathrm{U-MUSIC}}(z) = \tilde{\mathbf{a}}^T(1/z)\boldsymbol{E}_N\boldsymbol{E}_N^T\tilde{\mathbf{a}}(z) \tag{7}$$

where $\tilde{\mathbf{a}}(z) = \boldsymbol{Q}_M^H \boldsymbol{a}(z)$ with $z_i = e^{j2\pi d \sin\theta_i/\lambda}$ being the root of (7). Through finding the P roots which are closest to the unit circle, we determine the DOAs:

$$\theta_i = \sin^{-1}\left(\frac{\angle(z_i)\lambda}{2\pi d}\right), \quad i = 1, \cdots, P \tag{8}$$

where $\angle$ represents the angle operator.

It has been shown in [13] that the forward-backward (FB) and unitary root-MUSIC polynomials are identical. This in turn implies that the unitary root-MUSIC has the same performance as that of the FB root-MUSIC. But the unitary root-MUSIC method is much more computationally efficient since it is realized in terms of real-valued computation. However, the unitary root-MUSIC will suffer performance degradation, especially in the low SNR and small sample scenarios. This result is due to the fact that the unitary root-MUSIC estimator cannot efficiently handle the outliers. To circumvent this issue, we devise a PR based unitary root-MUSIC algorithm for computationally efficient and accurate DOA estimation.

III. Proposed Algorithm

Let us now formulate the PR based unitary root-MUSIC algorithm. Our aim is to remove the outliers and recover the approximate outlier-free performance of the unitary root-MUSIC by applying the PR technique. To this end, we resample the data matrix several times using synthetically generated pseudo-noise [6], [11]. For each PR process, we run the unitary root-MUSIC method to estimate the DOAs, and then utilize a reliability test to remove the outliers that are rejected by the test and only retain the remaining DOAs.

A. Pseudo-Noise Resampling Process

The key of the proposed method is to test the following hypothesis $\mathcal{H}$ for each DOA estimator [6], [10], [11], which enables us to separate the normal and abnormal DOA estimators.

$\mathcal{H}$: All the DOA estimates in a DOA estimator are localized in $\hat{\Theta}$.

Here, $\hat{\Theta}$ is the pre-estimated angular sectors of source localization. A simple way of determining $\hat{\Theta}$ is to use the conventional beamformer [10], [12], [14]. Let

$$\begin{aligned} \hat{\Theta} = &\left[\theta_1^{\max} - \theta_1^{\mathrm{left}}, \theta_1^{\max} + \theta_1^{\mathrm{right}}\right] \\ &\cup \cdots \cup \left[\theta_P^{\max} - \theta_P^{\mathrm{left}}, \theta_P^{\max} + \theta_P^{\mathrm{right}}\right] \end{aligned} \tag{9}$$

where $\theta_p^{\max}, (p = 1, \cdots, P)$ are the coordinates of the P highest peaks of the conventional beamformer output, θ_p^{left} and $\theta_p^{\mathrm{right}}$ are the left and right boundaries of the pth subinterval, and both of them can be chosen as angular distances between the maximum of the pth peak and the left/right neighbor point with 3 dB drop, respectively.

When the hypothesis $\mathcal{H}$ is rejected, the data matrix $\boldsymbol{X} = [\boldsymbol{x}(1), \cdots, \boldsymbol{x}(N)]$ will be resampled K times using synthetically generated pseudo-noise. The $M \times N$ resampled data matrix is given as $\tilde{\boldsymbol{X}} = \boldsymbol{X} + \boldsymbol{Y}$ where $\boldsymbol{Y}$ is the $M \times N$ pseudo-noise matrix with mean-zero and covariance matrix $\sigma_Y^2 \mathbf{I}_M$. Here, σ_Y^2 is the variance of the pseudo-noise and its value should be comparable with the variance of the measurement noise σ_n^2. It is shown in [6], [10], [11], [12] that we can estimate σ_Y^2 as $\hat{\sigma}_Y^2 = p \cdot \hat{\sigma}_n^2$ where $p \approx 1$ is a user-defined parameter and $\hat{\sigma}_n^2$ is the consistent estimate of σ_n^2 given by $\hat{\sigma}_n^2 = \frac{1}{M-P}\sum_{i=P+1}^M \hat{\lambda}_i$ with $\hat{\lambda}_1 \geq \cdots \geq \hat{\lambda}_M$ being the ordered eigenvalues of the real-valued covariance matrix $\hat{\boldsymbol{C}}$. Here, $\hat{\boldsymbol{C}} = \mathrm{Re}\{\boldsymbol{Q}_M^H \hat{\boldsymbol{R}} \boldsymbol{Q}_M\}$ is the estimated real-valued sample covariance matrix with $\hat{\boldsymbol{R}} = \boldsymbol{X}\boldsymbol{X}^H/N$ being the sample covariance.

B. DOA Selection Strategy and DDS

For each resampling run, we apply the unitary root-MUSIC to obtain a DOA estimator $\boldsymbol{\theta}$ which contains P DOA estimates and the corresponding root estimator $\boldsymbol{Z}$. Let the ith estimator be

$$\boldsymbol{\theta}^{(i)} = \left\{\hat{\theta}_1^{(i)}, \cdots, \hat{\theta}_P^{(i)}\right\}, \quad \boldsymbol{Z}^{(i)} = \left\{\hat{z}_1^{(i)}, \cdots, \hat{z}_P^{(i)}\right\} \tag{10}$$

where $\hat{\theta}_1^{(i)} \leq \cdots \leq \hat{\theta}_P^{(i)}$ are the P DOA estimates and $\hat{z}_p^{(i)}$ is the pth DOA root corresponding to $\hat{\theta}_p^{(i)}$. Assuming that after K PR runs, we have K estimators which are used to form the estimator bank

$$\mathcal{B}_\theta = \left\{\boldsymbol{\theta}^{(1)}, \cdots, \boldsymbol{\theta}^{(K)}\right\}, \quad \mathcal{B}_Z = \left\{\boldsymbol{Z}^{(1)}, \cdots, \boldsymbol{Z}^{(K)}\right\}. \tag{11}$$

Performing the reliability test $\mathcal{H}$ to $\mathcal{B}_\theta$ yields the following two subsets

$$\mathcal{B}_{\theta,1} = \left\{\tilde{\boldsymbol{\theta}}^{(1)}, \cdots, \tilde{\boldsymbol{\theta}}^{(J)}\right\}, \quad \mathcal{B}_{\theta,2} = \left\{\bar{\boldsymbol{\theta}}^{(1)}, \cdots, \bar{\boldsymbol{\theta}}^{(K-J)}\right\} \tag{12}$$

where $\mathcal{B}_{\theta,1}$ consists of J estimators that are successfully accepted by $\mathcal{H}$ and $\mathcal{B}_{\theta,2}$ contains the remaining $(K-J)$ estimators that are rejected by $\mathcal{H}$. In addition, we define

$$\tilde{\boldsymbol{\theta}}^{(i)} = \left\{\tilde{\theta}_1^{(i)}, \cdots, \tilde{\theta}_P^{(i)}\right\}, \quad i = 1, \cdots, J \tag{13}$$

to represent the ith estimators accepted by $\mathcal{H}$.

If $0 < J < K$, i.e., there is at least one estimator in $\mathcal{B}_{\theta,1}$, a reliable process to determine the final DOA estimator is to average the J successfully resampled estimators. Since the DOA estimates in each estimator are sorted in ascending order, we have $\tilde{\boldsymbol{\theta}} = \frac{1}{J}\sum_{i=1}^{J}\tilde{\boldsymbol{\theta}}^{(i)}$ where the pth DOA in $\tilde{\boldsymbol{\theta}} = \{\tilde{\theta}_1, \cdots, \tilde{\theta}_P\}$ is calculated as

$$\tilde{\theta}_p = \frac{1}{J}\sum_{i=1}^{J}\tilde{\theta}_p^{(i)}. \tag{14}$$

If $J = 0$, which means all the resampled estimators fail to pass $\mathcal{H}$, we propose a robust DDS to determine the final DOA estimates. It is worth mentioning that the DDS exploits the information contained in the root estimator. However, it should be noted that this information is ignored in [10] and only the median average method [6], [11] is used to determine the final DOA estimates from $\mathcal{B}_\theta$. Specific steps of the DDS are given as follows:

1) Divide $\mathcal{B}_Z$ into P subsets with each subset contains K roots corresponding to a common DOA. Then the ith subset can be expressed as

$$\mathcal{F}_i = \left\{\hat{z}_i^{(1)}, \cdots, \hat{z}_i^{(K)}\right\}, \quad i = 1, \cdots, P. \tag{15}$$

2) Calculate the modulus of each $\hat{z}_i$ in (15)

$$|\mathcal{F}_i| = \left\{|\hat{z}_i^{(1)}|, \cdots, |\hat{z}_i^{(K)}|\right\}, \quad i = 1, \cdots, P. \tag{16}$$

3) Find the maximum $|\hat{z}_i|$ and store its index, i.e.,

$$[|\hat{z}_i|_{\max}, I_i] = \max_{i=1,\cdots,P} |\mathcal{F}_i| \tag{17}$$

where $|\hat{z}_i|_{\max}$ is the maximum value of $|\mathcal{F}_i|$ and I_i is the corresponding index.

4) Substitute $\{\hat{z}_{i,\max}\}_{i=1}^{P}$ into (8) to obtain the final DOA estimates.

When $J = 0$, all the DOA estimators are stored in $\mathcal{B}_{\theta,2}$. The PR unitary ESPRIT algorithm [11] discards the whole estimator bank and utilizes the standard unitary ESPRIT to re-estimate the final DOAs. Therefore, it has lost all the information brought by the PR technique. In [10], the median average is utilized to divide the DOA estimator bank into P subsets with each subset containing K DOA estimates that are corresponding to a common DOA and sorted in descending order. Then the pth final DOA estimate can be determined by choosing the average of the two DOA estimates in the middle of the pth subset if K is even or choosing the median of the pth subset if K is odd. In [4], a conventional average method that just takes the average of pth subsets as the pth DOA estimate is suggested. Clearly, in each subset, there always exists a DOA estimate that is closest to the true DOA. However, both the conventional and the median average methods cannot determine such a relatively optimal DOA estimate. As a result, it may not guarantee that its final DOA estimates are closest to the true DOA values. On the contrary, the DDS can efficiently circumvent this problem. It is observed from (15)–(17) that for each subset, i.e., $\{\mathcal{F}_i\}_{i=1}^{K}$, the DDS always selects the root that is closest to the unit circle as the ith final DOA estimate. Thus, all the DOA estimates determined by the DDS are always the one closest to the true DOAs. The effectiveness of the DDS will be further verified in Section IV.

TABLE I
PSEUDO-CODE OF PROPOSED ALGORITHM

Step 1 Use unitary root-MUSIC estimator to obtain P DOA estimates by data matrix $\boldsymbol{X}$.

Step 2 Test the hypothesis $\mathcal{H}$ for this estimator.
- If $\mathcal{H}$ is accepted, terminate the algorithm.
- If $\mathcal{H}$ is rejected, utilize PR process to generate K resampled data matrices, and then apply unitary root-MUSIC to form the estimator bank of (11).

Step 3 Apply reliability test to each resampled estimator from the estimator bank.
- If there are any J $(0 < J \leq K)$ estimators pass $\mathcal{H}$, then estimate the ith DOA θ_i via (14).
- If all DOAs are rejected by $\mathcal{H}$, then apply DDS to estimate the DOAs.

Note that the computational complexity of the proposed algorithm is higher than the unitary root-MUSIC by a factor $(1 + \rho K)$, where ρ is the probability of outliers, i.e., the probability that the unitary root-MUSIC estimator may be rejected by $\mathcal{H}$. The additional computational burden improves the performance in return. Due to the low complexity of unitary root-MUSIC, the proposed algorithm can be implemented efficiently when a proper K is given.

IV. SIMULATIONS

We compare the proposed method with the root-MUSIC, unitary root-MUSIC, PR root-MUSIC [10] and PR unitary ESPRIT [11] algorithms in terms of root mean square error (RMSE). In order to illustrate the effectiveness of the DDS, we add another DDS based algorithm for comparison, which is obtained via replacing the median average method in the PR root-MUSIC with the proposed DDS, i.e., "PR root-MUSIC with DDS". Meanwhile, we examine their ability of removing outliers as well, namely, the probability of outliers. The CRLB is plotted as a benchmark. In our simulations, two independent narrowband Gaussian signals are assumed to imping upon a ULA with $M = 8$ omnidirectional sensors from directions $\theta_1 = 2^\circ$ and $\theta_2 = 7^\circ$. The noise is zero-mean white Gaussian process. The SNR is defined as the ratio of the power of all source signals to that of the additive noise at each sensor. Furthermore, the user defined parameter p is set to be 1 in all the simulations. 1000 Monte Carlo simulations are carried out to evaluate the RMSE, which is defined as $\mathrm{RMSE} = \sqrt{\frac{1}{1000P}\sum_{i=1}^{P}\sum_{j=1}^{1000}\left(\hat{\theta}_{i,j} - \theta_i\right)^2}$. In all experiments, we assume that the source localization sectors and the number of sources are known or estimated by [15], [16]. According to (9), we use the conventional beamformer to pre-estimate $\hat{\Theta}$ in each independent run.

In the first simulation, we study the ability of removing outliers under small sample size as a function of SNR. We set the

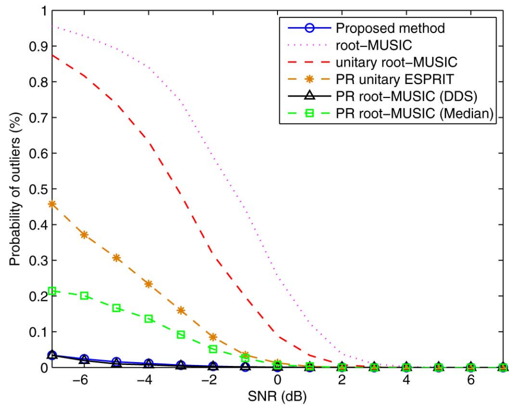

Fig. 1. Probability of outliers versus SNR. ($N = 30$).

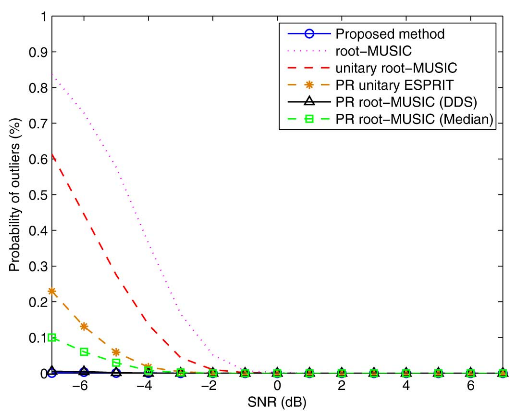

Fig. 2. Probability of outliers versus SNR. ($N = 100$).

number of PR runs as $K = 15$. Figs. 1 and 2 show the outlier probability for $N = 30$ and $N = 100$, respectively. It is seen that the proposed algorithm provides a considerable improvement to remove the outliers. When $N = 100$, the proposed algorithm has almost removed all the outliers. However, the other DOA estimation algorithms suffer a high probability of outliers no matter N is small or large. Moreover, we find that the DDS based PR root-MUSIC has achieved almost the same outlier removing performance compared to the median average based PR root-MUSIC algorithm [10].

In the second example, we set $\sigma_n^2 = 1$ and vary the signal power such that the input SNR goes from -7 to 7 dB. The number of snapshots is $N = 100$. We examine the performance with different numbers of PR processes. For the PR root-MUSIC and PR unitary ESPRIT algorithms, we set $K = 15$. The RMSEs are depicted in Fig. 3. It can be seen that in low SNR regime, the proposed algorithm outperforms the conventional and unitary root-MUSIC schemes no matter how small or large the K is. Fig. 3 also implies that a relatively larger K can lead to a better performance for the proposed method and the value of $K = 15$ is sufficient to obtain a satisfactory performance. When the SNR is larger than 3 dB, the proposed and unitary root-MUSIC algorithms converge together. This is due to the fact that, at high SNRs, all the DOA estimates are accepted by the hypothesis test, and the proposed one is reduced to be the unitary root-MUSIC scheme.

Let us now study the RMSE versus the sample size. In this example, we fix the SNR to -2 dB. For all the PR based algorithms, we set the number of PR runs to be $K = 15$ for fair

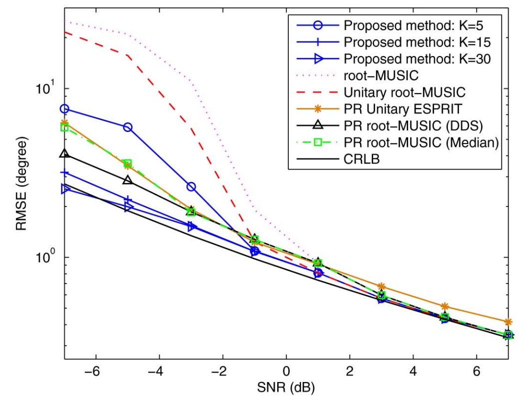

Fig. 3. RMSE angle error performance versus SNR. ($N = 100$).

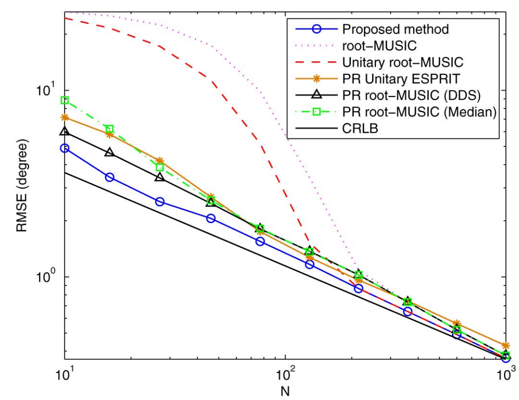

Fig. 4. RMSE angle error performance versus N. ($\mathrm{SNR} = -2$ dB).

comparison. It is observed from Fig. 4 that the proposed approach performs the best. When $N < 100$, the proposed PR unitary root-MUSIC algorithm achieves a better performance improvement compared to the unitary root-MUSIC scheme. From Figs. 3 and 4, we find the DDS-based PR root-MUSIC algorithm always outperforms the median average based one, especially at low SNRs and small samples.

V. CONCLUSION

A PR based unitary root-MUSIC algorithm has been derived for DOA estimation. In contrast to the existing root-MUSIC methods, the proposed scheme combines the unitary root-MUSIC and PR techniques to form a DOA estimator bank and a corresponding root estimator bank. Meanwhile, a DOA selection strategy with the DDS is proposed to select the reliable DOA estimates. Here, the DDS which exploits the DOA roots information inherent in the unitary root-MUSIC algorithm can help to determine the final DOA estimates when all the DOA estimators are rejected by the reliability test. Simulation results verify the effectiveness of the proposed algorithm.

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